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U. S. NAVAL AIR DEVELOPMENT CENTER

JOHNSVILLE, PENNSYLVANIA

Aviation Medical Acceleration Laboratory

NADC-MA-6216

26 December 1962

Flame Contact Studies

II. Experimental Validation of Mathematical
Analysis of Heat Flow

Bureau of Medicine and Surgery

Subtask MR005.13-1005.1

Report No. 29

Bureau of Naval Weapons

WepTask RAE 20J 010/2021/F012 10 002

Problem Assignment No. J44AE22-2



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SUMMARY

Mathematical equations for transient heat flow through a two-layer wall are examined, corrected, and validated experimentally for application to thin layers of material (0 to 1 mm thick) during short-term heating (3 sec) in gas flames.

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INTRODUCTION

An apparatus and a method for the study of heat transfer through fabrics in contact with gas flames at 1200°C have been described (1). Data obtained directly from experimental observations were given for destruction temperature, resistance to heat transfer, and the insulation effect of air spaces between layers of fabrics woven of a new high-temperature-resistant fiber (Du Pont HT-1). The present report concerns the establishment of a system for the mathematical analysis of these experimental data whereby surface and interface temperatures, temperatures in depth, and various properties of the fabrics may be derived. The equations examined are those for transient heat flow through a two-layer wall as proposed by Griffith and Horton (2).

PROCEDURE AND MATERIALS

Due to the total dependence to be placed upon the mathematical expressions chosen to represent the heat flow situation, it was necessary to validate the analytical process experimentally. The validation required a substance of known thermal properties that could be obtained in fairly thin sheets, would not wrinkle or swell appreciably on flame contact, and would form a tight contact with the backing material. Silicone rubber* was selected for this purpose. Bubble-free sheets of this material were molded

* RTV-20 manufactured by the General Electric Company

on a plastic surface in thicknesses from about 0.5 to 1.0 mm. The plastic imparted a smooth surface to the rubber which, due to its surface properties, formed an intimate contact with the backing material without the use of a filming or filling agent. (The backing material was the simulated skin device made by the Naval Material Laboratory, N.Y., and previously described[3].)

Exposures of the backed silicone rubber were made until the temperature rise at a fixed time during exposure, at a fixed depth beneath the surface of the simulated skin, was established for several thicknesses of the rubber. In the equations of Griffith and Horton the expression for this temperature rise was given as (Equation 1, Appendix):

$$U_2 = \frac{2H\lambda\sqrt{D_1}}{\gamma} \sum_{n=0}^{\infty} \left(-\frac{1}{\gamma}\right)^n \left\{ 2\sqrt{\frac{D_2 t}{\pi}} e^{-\left\{x - \alpha\left\{1 - \sqrt{D_2/D_1}(2n+1)\right\}\right\}^2/4D_2 t} - \left[x - \alpha\left\{1 - \sqrt{D_2/D_1}(2n+1)\right\}\right] \left(1 - \operatorname{erf} \frac{x - \alpha\left\{1 - \sqrt{D_2/D_1}(2n+1)\right\}}{2\sqrt{D_2 t}}\right) \right\}$$

where

subscript 1 refers to the outer layer (silicone rubber)

subscript 2 refers to the base layer (simulated skin)

and

U = temperature rise

H = heat flux perpendicular to surface

x = total thickness from surface to point of temperature rise measurement

α = thickness of layer 1

D = thermal diffusivity = k/S

k = thermal conductivity

S = volume specific heat (density x specific heat)

$$\gamma^* = \frac{k_2 S_2 + \sqrt{k_1 S_1 k_2 S_2}}{k_2 S_2 - \sqrt{k_1 S_1 k_2 S_2}}$$

$$\lambda = (k_2 \sqrt{D_1} - k_1 \sqrt{D_2})^{-1}$$

t = time

*As corrected

As seen from this equation, in addition to the thermal constants for both layers, the heat flux and the thickness of the layers must be known in order to compare the computed value with the experimental value. The heat flux was measured directly by means of a flame impingement calorimeter** (4) in the classical manner of water calorimetry as described in detail elsewhere (5). This instrument had been designed to measure much higher fluxes so that very low rates of flow had to be maintained in order to produce an appreciable temperature difference between the incoming and outgoing water. Elimination of bubbles in the water system was difficult but was aided by addition of a small amount of wetting agent (5 drops of a 10% solution of benzalkonium chloride per liter of reservoir water). Routinely, the calorimeter was placed with the sensing element at the level of the experimental specimen and heated until equilibrium was reached as indicated by a constant temperature difference between the inflow and outflow water. The outflow was collected over a carefully clocked

** The loan of this instrument through the courtesy of D.L. Richardson of the Arthur D. Little Company is very gratefully acknowledged.

period of about 10 seconds and weighed. The rate of flow was then determined and the heat flux calculated from the relationship:

$$H = \frac{^{\circ}\text{C} \times \text{cc/sec}}{1.268}$$

where H = heat flux ($\text{cal/cm}^2 \text{ sec}$)

$^{\circ}\text{C}$ = temperature difference between inflow and outflow

cc/sec = rate of flow

1.268 = area of sensing element (cm^2)

The heat flux was determined, also, indirectly by the temperature rise at depth in a substance of known thermal properties, the simulated skin. In this procedure the bare simulated skin was exposed to the flame for 3 seconds and the temperature rise at depth noted. Then the heat flux was determined from the relationship between temperature rise at a known depth and heat flux perpendicular to the surface (5).

$$H = \frac{\Delta T_o}{36.2}$$

where H = heat flux in $\text{cal/cm}^2 \text{ sec}$

ΔT_o = temperature rise at depth 0.049 cm at 3 sec in $^{\circ}\text{C}$

36.2 = temperature rise in $^{\circ}\text{C}$ /unit heat flux given for the simulated skin (3) and also calculable by Eq. 1 when $\alpha = 0$.

The thickness of the silicone rubber slabs was determined from sections carefully cut from the area which had been located above the embedded thermocouple during the heat exposures. Some measurements were made with a

$$\gamma^* = \frac{k_2 S_2 + \sqrt{k_1 S_1 k_2 S_2}}{k_2 S_2 - \sqrt{k_1 S_1 k_2 S_2}}$$

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skin (3) and also calculable by Eq. 1 when $\alpha = 0$.

The thickness of the silicone rubber slabs was determined from sections carefully cut from the area which had been located above the embedded thermocouple during the heat exposures. Some measurements were made with a

dissecting microscope on unmounted sections and others were made at higher magnifications on paraffin-mounted sections. **

For short exposure times, the simulated skin provided an essentially semi-infinite base layer. The embedded thermocouple was 0.001 cm thick and was situated at a level 0.050 cm beneath the surface of the simulated skin; thus, for purposes of calculation, the thickness of the second layer was 0.049 cm.

Of the material constants required for application of the equations, only the density was corroborated by measurements made in this laboratory; the diffusivity, volume specific heat, and thermal conductivity values used were those supplied by the manufacturers or computed from the data supplied.

EXPERIMENTAL DATA AND DISCUSSION

The pertinent physical and thermal properties of the silicone rubber and of the simulated skin are shown in Table I.

** The kind assistance of Wade Border in preparing these sections is acknowledged with much gratitude.

TABLE I

Property	Units	Silicone Rubber	Simulated Skin
Thickness	cm	0.050 - 0.100	1.0
Density	gm/cm ³	1.35	1.82
Vol. specific heat	cal/cm ³ °C	0.47	0.65
Diffusivity	cm ² /sec	13.5×10^{-4}	20.3×10^{-4}
Thermal conductivity	cal/cm °C sec	6.4×10^{-4}	31.1×10^{-4}

Measurements of the heat flux by means of the flame impingement calorimeter proved to be quite variable as might be expected since the instrument had been designed to measure flow levels fifty times as high as those concerned here. The calorimeter water flow ranged from 0.20 to 0.47 cc/sec and the temperature difference, from 3.46 to 10.42°C. The average of thirty apparently reliable measurements was 1.61 cal/cm² sec \pm 0.25, gross deviation with a standard deviation (σ) of \pm 0.12°C. Since all systematic errors would be in the direction of lowering this value, the ten highest measurements were averaged separately. These yielded a value of 1.74 cal/cm² sec \pm 0.12, gross deviation.

Ten measurements of heat flux made with the simulated skin yielded an average value of 1.73 cal/cm² sec \pm 0.02, gross deviation. This method proved to be far more convenient, sensitive and accurate than

the calorimetric procedure at this level of flux. Therefore, the value of $1.73 \text{ cal/cm}^2 \text{ sec}$ was accepted as correct while the calorimeter data were considered to be corroborative inasmuch as the average value of the simulated skin measurements was in excellent agreement with the average of the ten highest calorimetric determinations, and within one standard deviation of the average of all thirty apparently reliable determinations.

Temperature rise at depth was measured routinely at the exposure time of 3 seconds. Measurements were made under slabs of silicone rubber of three different thicknesses. At the same time computations of the temperature rise at depth, at the interface, and at the surface were made for the appropriate thicknesses of the two layers. It was found that the equations given yielded a slightly higher temperature rise at depth than at the interface, an obvious physical impossibility; therefore, a proof of the derivation was performed.*

The analysis showed that the equations were correct but that the term, γ , was incorrect. Instead of:

$$\gamma = \frac{k_2 S_2 + k_1 S_1}{k_2 S_2 - k_1 S_1}$$

* For this analysis and the consequent corrections, the authors are most grateful to Eugene Toll of the Aeronautical Computer Laboratory, U. S. Naval Air Development Center, Johnsville, Pa.

this value should have been:

$$\gamma = \frac{k_2 S_2 + \sqrt{k_1 S_1 k_2 S_2}}{k_2 S_2 - \sqrt{k_1 S_1 k_2 S_2}}$$

With the incorporation of this correction the theoretical temperature rise at the interface became appreciably higher than that at depth as expected. It was then possible to compare the experimental results with the theoretical values.

Table II presents this comparison for the temperature rise measured within the backing layer and the values obtained by the corrected Equation (1) using the measured flux of 1.73 cal/cm² sec.

TABLE II

Thickness of Rubber Layer (mm)	No. of Measurements	Temperature Rise (°C)	
		Measured ± σ	Theoretical (°C/H x 1.73)
0.95	17	10.98 ± 0.40	10.90
0.55	12	28.13 ± 0.84	28.20
0.52	13	30.09 ± 0.55	29.93

It is seen that these values are in excellent agreement, the averages of the experimental results being almost identical with the theoretical values.

The complete analysis of the temperature relationships is illustrated in Figure 1 in which the temperature rises at depth within the simulated

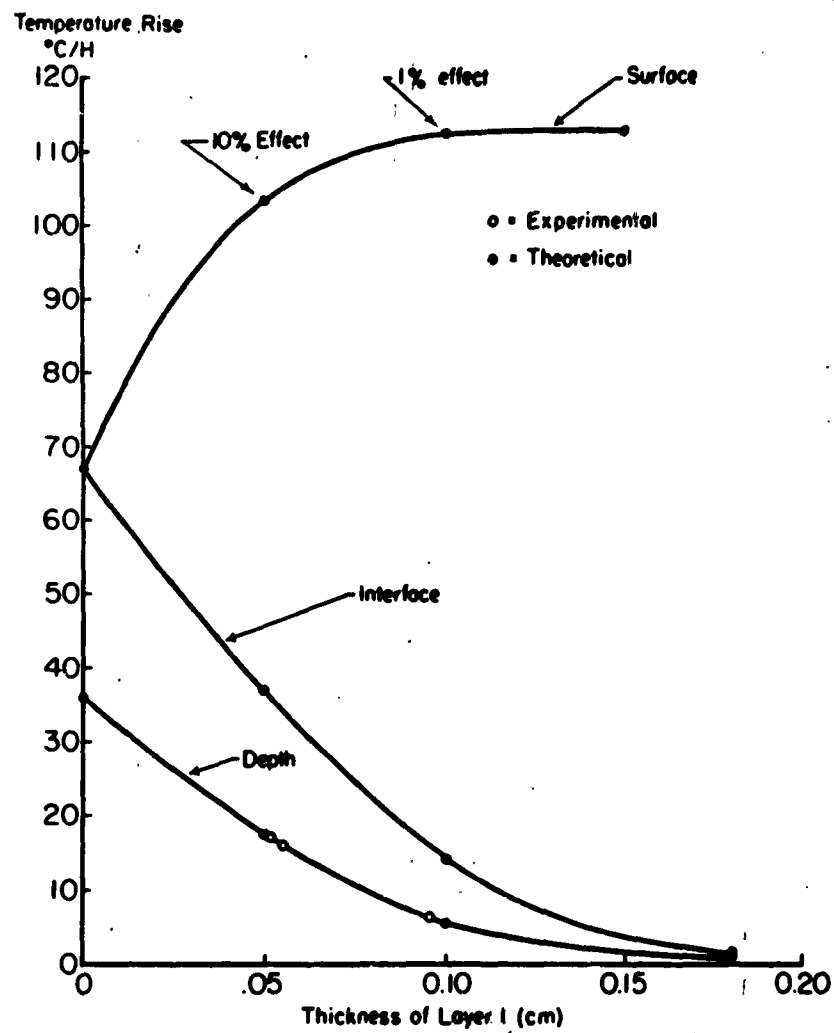


Figure 1. Temperature rise per unit heat flux at surface, interface and depth within layer 2 (simulated skin) and comparison of experimental and theoretical data.

skin, at the interface, and at the surface of the silicone rubber are plotted against the thickness of the rubber layer (α) for the exposure time of 3 seconds. The comparison of experimental and theoretical data is shown graphically in the bottom curve, that of temperature rise at depth. From the surface temperature rise curve it is seen that the "critical thickness" occurs at about 0.1 cm. At greater thicknesses the temperature rise as obtained from Eq. 2 coincides precisely with the values obtained from Eq. 4, the well-known expression for the temperature rise of the surface of a homogeneous wall. At lesser thicknesses the effect of the underlying layer is obvious in the depression of the surface temperature rise. Extrapolation of this curve to zero thickness yields the surface temperature rise of the simulated skin material as given (3, 5) and as may be calculated from Eq. 4 on substitution of the thermal properties of this layer. Furthermore, during the check on the derivation of the equations, it was found that the surface temperature rise of the bare surface of the second layer could be obtained from the surface temperature rise of the first layer at critical thickness or greater if the thermal properties of both layers are known. This end could be accomplished by inclusion in Eq. 4 of the next term of the expansion of Eq. 3, viz., a multiplier $\left(\frac{\gamma - 1}{\gamma + 1} \right)$, expressive of the ratio of the thermal properties of the two layers. Thus, for these materials, where

$$0.598 = \frac{\gamma - 1}{\gamma + 1}$$

and $112.4^{\circ}\text{C}/\text{H}$ = temperature rise of silicone rubber surface at critical thickness or greater from both Eq. 2 and Eq. 4, then $112.4 \times 0.598 = 67.2^{\circ}\text{C}/\text{H}$, the temperature rise of the bare simulated skin surface as given and as calculated from Eq. 4 on substitution of the properties of the simulated skin itself. Finally, the curve for the interface temperature rise, an extrapolation to zero thickness, passes through this point also, completing the confirmation of total internal agreement among the mathematical expressions.

SUMMARY AND CONCLUSION

Experimental validation studies of the equations of Griffith and Horton for transient heat flow through a two layer assembly (2) were conducted with thin silicone rubber slabs overlaid on a simulated skin of known thermal properties. This study resulted in important revisions in the equations by correction of an error in the derivation of one of the constants embodying the thermal properties of materials of both layers. With the incorporation of these corrections, complete agreement was attained between theoretical and experimental values and between values computed by two different forms of the basic derivation.

It is concluded, therefore, that these corrected equations comprise a rigorous representation of the thermal events and may be used with confidence in solving problems which satisfy the boundary conditions specified. In particular, these equations are applicable to the immediate

problem for which they were sought, viz., studies of temperature rise under experimental fabrics on flame contact, and of the thermal properties of these fabrics. It is expected, furthermore, that they will be of inestimable value in solving physiological problems of heat flow into living skin where temperature at a shallow depth cannot be measured accurately.

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APPENDIX I

Equation 1 - TEMPERATURE RISE AT DEPTH IN LAYER 2

$$U_2 = \frac{2H\lambda\sqrt{D_1}}{\gamma} \sum_{n=0}^{\infty} \left(-\frac{1}{\gamma}\right)^n \left\{ 2\sqrt{\frac{D_2}{D_1}} e^{-\left\{x-a\right\}\sqrt{D_2/D_1}(2n+1)} \right\}^2 / 4D_2 t$$

$$- \left\{x-a\right\}\sqrt{D_2/D_1}(2n+1) \left\{1 - \operatorname{erf} \frac{x-a\left\{1-\sqrt{D_2/D_1}(2n+1)\right\}}{2\sqrt{D_2 t}} \right\} \right\}$$

where

subscript 1 refers to top layer 1, subscript 2 refers to base layer 2
and U = Temperature rise

H = Heat flux perpendicular to surface

X = Total thickness from surface to point of temperature
rise measurement

a = Thickness of layer 1

D = Thermal diffusivity = k/s

k = Thermal conductivity

S = Volume specific heat (density x specific heat)

$$\gamma = \frac{k_2 S_2 + \sqrt{k_1 S_1 k_2 S_2}}{k_2 S_2 - \sqrt{k_1 S_1 k_2 S_2}}$$

$$\lambda = (k_2 \sqrt{D_1} - k_1 \sqrt{D_2})^{-1}$$

t = time

*As corrected

APPENDIX I

Equation 2 - TEMPERATURE RISE AT SURFACE

$$U_0 = \frac{H}{k_1} \left[2\sqrt{\frac{D_1 t}{\pi}} - \sum_{n=0}^{\infty} \left(-\frac{1}{\gamma}\right)^n \left\{ \sqrt{\frac{D_1 t}{\pi}} e^{-a^2 \frac{(n+1)^2}{4D_1 t}} - a(n+1) \left(1 - \operatorname{erf} \frac{a(n+1)}{\sqrt{D_1 t}}\right) \right\} \right]$$

Equation 3 - TEMPERATURE RISE AT INTERFACE

$$U_0 = \frac{H}{k_1} \sum_{n=0}^{\infty} \left(-\frac{1}{\gamma}\right)^n \left(-\frac{1}{\gamma}\right) \left[2\sqrt{\frac{D_1 t}{\pi}} e^{-\left(\frac{a(2n+1)}{2\sqrt{D_1 t}}\right)^2} - a(2n+1) \left(1 - \operatorname{erf} \frac{a(2n+1)}{2\sqrt{D_1 t}}\right) \right]$$

Equation 4 - TEMPERATURE RISE AT SURFACE OF SEMI-INFINITE HOMOGENEOUS SOLID

$$U_0 = 2H\sqrt{(1/S_1 k_1 \pi)} = (2H/k_1)\sqrt{(D_1 t/\pi)}$$

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